The Intermediary Asset Pricing Approach to Puzzles in Macrofinance*

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Abstract

This article introduces a model with high time-varying volatility in consumption. The model explains a series of puzzles in macrofinance, including the (i) equity premium puzzle, (ii) the risk-free rate puzzle, (iii) the bond premium puzzle, and (iv) the predictability of aggregate stock market returns with price-dividend ratios. Furthermore, it is argued that stochastic volatility of the kind introduced in this article is a necessary ingredient for explaining these puzzles within a broad class of models.

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1 Introduction

A series of asset pricing puzzles have been documented and studied within macrofinancial models. These models typically rely on a representative consumer. However, as indicated by various strands of literature, the marginal investor likely differs from the representative household. First, it has long been known that regular households have limited stock market participation (for example Haliassos and Bertaut 1995; Campbell 2006). Next, there is a supply and demand driven approach to asset pricing, whose success shows that agents, such as *arbitrageurs*, have a special role in the pricing process (for example Vayanos and Vila 2021). In addition, the intermediary asset pricing approach takes this further, and uses financial intermediaries as marginal investors (for example He and Krishnamurthy 2013).

In the current article, intermediaries are modelled as consumers with an exogenous consumption process, and it is shown that an explanation of basic asset pricing puzzles arises naturally. The main ingredient offered by the intermediary perspective is the high consumption volatility of the investor. Intuitively, consumption volatility can become large because intermediaries can get highly exposed to a particular source of risk when it associated with especially lucrative investment opportunities. This channel can yield positive risk premia on both real bonds and equity. Next, the intermediary perspective also provides a rationale, explaining why not just stocks but also bonds have a positive risk premium. In particular, this is related to the fact that the wealth of intermediaries can rise when interest rates fall, making bonds perform well in good times and badly in bad times. This makes bonds genuinely risky for marginal investors, and for that reason they command a positive premium.

Most asset pricing puzzles are associated with statistics observed in the data, which are not easily explained by baseline macroeconomic models. In addition, these statistics are related to central economic variables that are relevant for both asset pricing and the macroeconomy. The issue arises, because the models typically generate corresponding statistics, that are either qualitatively different or significantly different in magnitude than the data. The puzzles that are addressed in this article have been demonstrated in the literature.[1] The equity premium puzzle was first introduced by Mehra and Prescott (1985), who showed that excess returns on the stock market can't be explained with moderate values of risk aversion by the standard consumption-based model, when calibrated using aggregate consumption

[1]A good summary of these puzzles is provided in Gabaix (2012).

data. In addition, trying to explain the equity premium puzzle with high risk aversion leads to an extremely variable risk-free rate, which is also inconsistent with the data, and is referred to as the risk-free rate puzzle (Weil 1989). Apart from stocks, long-term bonds also exhibit positive excess returns, which are large and variable in time, while most standard models predict excess returns for bonds that are negative and small in absolute value. This is the bond premium puzzle (Backus, Gregory and Zin 1989).[2] Originally, it was introduced for nominal bonds, but real bonds also have the same critical features (Abrahams, Adrian, Crump, Moench and Yu 2016; d' Amico, Kim and Wei 2018; Pflueger and Viceira 2016). While the bond premium puzzle usually refers to the difficulty of matching both the level of excess premia and their variability/predictability, the equity premium puzzle only refers to the level of equity premia. Nevertheless, aggregate stock market returns are not only high but they are also predictable from price-dividend ratios (Campbell and Shiller 1988). In the current article, by using the intermediary perspective, all of these puzzles are naturally explained. Indeed, it is argued that a high consumption volatility is part of almost all explanations of these puzzles. For example, limited stock market participation (Basak and Cuoco 1998), intermediary asset pricing (He and Krishnamurthy 2013), and supply and demand driven explanations (Vayanos and Vila 2021) all generate marginal investors with high time-varying volatility in consumption (or wealth). Especially for the bond premium puzzle, the only alternative to a high consumption volatility is a high time-varying risk aversion, such as in habit models (Campbell and Cochrane 1999; Wachter 2006).[3]

In the model, there is one state variable that simultaneously and positively affects the consumption drift and diffusion. Intuitively, this state variable reflects investment opportunities, that are available to the marginal investor. Crucially, these investment opportunities do not arise due to changes in fundamentals, but they arise because interest rates are changing. When the value of the state variable is high, investment opportunities are favorable. By pursuing these opportunities, the marginal investor also increases the risk of her portfolio, which leads at the same time to a higher consumption volatility, and a higher consumption drift. Given a high time-varying consumption volatility of the marginal investor, risk premia also become high in absolute value and time-varying. This provides an explanation for the equity premium puzzle. In addition, as changes in consumption

[2] The bond premium puzzle in the literature often refers to term premia. These are excess returns of long-term bonds over short-term bonds for an investment horizon which is equal to the maturity of the long-term bond, and they are defined precisely later in the article.

[3] It is also possible to have explicit time-varying risk aversion, such as in Lettau and Wachter (2011).

volatility explain risk premia, they become an extra source of price volatility, and by affecting price-dividend ratios they also induce predictability. This provides an explanation for the apparent predictability of excess returns.

Furthermore, different levels of investment opportunities correspond to different levels for the risk-free interest rate and different levels of marginal investor wealth. This is consistent with stochastic changes in the state variable being associated with stochastic changes in marginal investor consumption.[4] In particular, the model is constructed and calibrated, so that as investment opportunities improve, the risk-free interest rate falls, and the marginal investor's wealth and consumption rises. This implies that long-term bonds carry a positive risk premium, given that their high performance is associated with good states of the world for the marginal investor, thus explaining the bond premium puzzle. It may seem counterintuitive that the risk-free rate falls when investment opportunities improve. However, this is possible, and not that surprising given that the state variable is not strongly associated with changes in firms' fundamentals. Thus, when the instantaneous risk-free rate falls, it is easier, for example, to borrow and invest in riskier assets, such as long-term bonds and equity.[5] The intuition here is similar to the intuition described in Vayanos and Vila (2021). Indeed, that article also comments on the fact that changes in interest rates, such as the ones caused by central banks, can be a source of income for arbitrageurs, which could be disconnected from fundamentals. The model does not include inflation, and all quantities are real.

Section 2 introduces the model, Section 3 discusses the calibration, Section 4 presents the results, Section 5 discusses the special case of recursive preferences, and Section 6 concludes.

2 Model Setup

2.1 One Marginal Investor

Similar to He, Kelly and Manela (2017), the model abstracts from non-marginal investors, and focuses on one kind of agent, who is also the marginal investor. Theoretically, this could be the only agent in the economy, but it is more realistic to think that other agents also exist but do not participate in the same asset market due to frictions. While it makes more sense to think of the marginal investors

[4] In other words, the noise processes of consumption and the state variable are correlated.

[5] Under a different calibration, an improvement in fundamentals could lead to an increase in the risk-free rate.

as some kind of financial intermediary, it is also possible to imagine that a small group of households simultaneously participate in financial markets and have a high consumption volatility. For example, Ait-Sahalia, Parker and Yogo (2004) show evidence that high net-worth households have a much higher consumption volatility than regular households.

2.2 Utility

The lifetime utility of the agent is given by:

$$U_0 = \mathrm{E}_0 \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma} - 1}{1-\gamma} \mathrm{d}t, \tag{1}$$

where C_t is the consumption flow at time t, γ is the coefficient of relative risk aversion, and ρ is the time preference rate. In the model, time is continuous, utility is time-separable, and the utility flow exhibits constant relative risk aversion (CRRA).

2.3 Consumption Process

The consumption flow C_t is exogenous and follows a stochastic process:

$$\begin{aligned} \mathrm{d}\log(C_t) = \mathrm{d}c_t &= \mu_{ct}\mathrm{d}t + \sigma_{ct}\mathrm{d}W_{ct} \\ \mu_{ct} &= \mu_{c0}\exp(x_t)^{\zeta} \\ \sigma_{ct} &= \sigma_{c0}\exp(x_t)^{\eta} \end{aligned} \tag{2}$$

where c_t is log consumption, x_t is the state variable, W_{ct} is a Wiener process associated with consumption, μ_{ct} is the consumption drift, σ_{ct} is the consumption diffusion, μ_{c0} is the drift of consumption at the steady state, σ_{c0} is the steady state consumption diffusion, and ζ and η are parameters that govern the dependence of the consumption process on the state variable. Both ζ and η are positive, which implies that both the consumption drift and the consumption diffusion are increasing in the state variable x_t. The most important feature of the consumption process is its high time-varying volatility. As is shown later, this can generate a high time-varying equity premium and also a high (at least in absolute value) time-varying bond premium.

The drift of the consumption process is also assumed to be increasing in the state variable. This assumption provides a rationale for the time-varying volatility, as the marginal investor increases the risk of her portfolio when there are better investment opportunities. The better investment opportunities induce a higher consumption drift, and the higher risk increases the volatility of consumption. In order for bond premia to also be positive, the interest rate must also co-vary negatively with consumption. This implies that the price of bonds co-varies positively with consumption, which makes the bonds risky for the marginal investor.

2.4 State Variable

There is only one state variable x_t, and it follows a mean reverting process:[6]

$$\begin{aligned} \mathrm{d}x_t &= \log(\phi)x_t\mathrm{d}t + \sigma_{xt}\mathrm{d}W_{xt} \qquad (3) \\ \mathrm{d}W_{xt}\mathrm{d}W_{ct} &= \rho_{cxt}\mathrm{d}t \end{aligned}$$

where σ_{xt} is the state variable diffusion, W_{xt} is a Wiener process associated with the state variable, and ρ_{cxt} is the correlation between the noise processes for consumption and the state variable. The drift term shows that the steady state is at $x_t = 0$, and the process always reverts to the steady state given that $0 < \phi < 1$.[7]

2.5 Stochastic Discount Factor

Based on the utility function the stochastic discount factor (SDF) is given by:

$$\Lambda_t = e^{-\rho t}C_t^{-\gamma} \qquad (4)$$

Then based on the consumption process, and by applying Itô's lemma, the dynamics of the SDF can be derived:[8]

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = \Big(-\rho - \gamma\mu_{ct} + \frac{\gamma^2}{2}\sigma_{ct}^2\Big)\mathrm{d}t - \gamma\sigma_{ct}\mathrm{d}W_{ct} \qquad (5)$$

2.6 Instantaneous Risk-Free Rate

Based on the SDF the instantaneous risk-free rate can be derived:

$$r(x_t)\mathrm{d}t = -\mathrm{E}_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\right] = \Big(\rho + \gamma\mu_{ct} - \frac{\gamma^2}{2}\sigma_{ct}^2\Big)\mathrm{d}t \qquad (6)$$

[6] ϕ corresponds to the coefficient of an AR(1) process in discrete time.
[7] This is similar to an AR(1) where $x_{t+1} = \phi x_t + \epsilon_t$
[8] The pricing follows the notation and the approach of Cochrane (2009).

2.7 Zero-Coupon Bond Pricing

Then, zero-coupon bonds that mature at time $T = t + m$ are priced using the pricing equation:

$$\mathrm{E}\big[\mathrm{d}(\Lambda_t B_t(x,m))\big] = 0 \tag{7}$$

where $B(x,m)$ is the price of a zero-coupon bond with a remaining maturity of m at time t. The price of the bond is a function of the state variable x_t, and maturity m. Using Itô's lemma on the bond price, yields the following:

$$\mathrm{d}B_t = \left(\log(\phi)x_t B_x - B_m + \frac{\sigma_{xt}^2}{2}B_{xx}\right)\mathrm{d}t + \sigma_{xt}B_x\mathrm{d}W_{xt}, \quad B(x,0) = 1 \tag{8}$$

where subscripts on B denote partial derivatives with respect to the corresponding variable.[9] Using the process for the bond price, the pricing equation gives rise to a partial differential equation (PDE) (time subscripts and the arguments of B are omitted for brevity):

$$\begin{aligned} &\mathrm{E}\big[\mathrm{d}(\Lambda B)\big] = 0 \Rightarrow \mathrm{E}\left[\frac{\mathrm{d}\Lambda}{\Lambda}B + \mathrm{d}B + \frac{\mathrm{d}\Lambda}{\Lambda}\mathrm{d}B\right] = 0 \\ &\Rightarrow -B_m - r(x_t)B + \big(\log(\phi)x_t - \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\big)B_x + \frac{\sigma_{xt}^2}{2}B_{xx} = 0 \end{aligned} \tag{9}$$

where B_m is the derivative of the bond price with respect to maturity, B_x is the derivative of the bond price with respect to the state variable, and B_{xx} is the second derivative of the bond price with respect to the state variable. This differential equation can be solved numerically. In particular, the Feynman-Kac formula as shown in Appendix A. Based on the expressions above the instantaneous expected excess return of the bond before maturity can be derived as:

$$\begin{aligned} \mathrm{E}_t\left[\frac{\mathrm{d}B}{B}\right] + \mathrm{E}_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\right] = \underbrace{\mathrm{E}_t\left[\frac{\mathrm{d}B}{B}\right] - r\mathrm{d}t}_{\equiv\text{Expected Excess Return}} = -\mathrm{E}_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\frac{\mathrm{d}B}{B}\right] \\ \Rightarrow -\frac{B_m}{B} + \log(\phi)x\frac{B_x}{B} + \frac{\sigma_{xt}^2}{2}\frac{B_{xx}}{B} - r = \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\frac{B_x}{B} \end{aligned} \tag{10}$$

[9] For simplicity the dependence on time is not explicitly shown, but maturity is connected to time through $\partial/\partial t = -\partial/\partial m$.

And the term premium of the bond can be defined as:[10]

$$TP(x_t, m) = -\frac{1}{m} \log \left[\frac{B(x_t, m)}{\mathrm{E}_t \left[\exp \left\{ -\int_0^m r(x_{t+s}) \mathrm{d}s \right\} \right]} \right] \tag{11}$$

Intuitively, this expresses the expected annualized excess return of the bond over a period equal to its remaining maturity. The result in Equation (10) is interesting because it provides a way to measure the excess return of the bond. In addition, based on Equation (9) $\rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}$ can be compared to $\log(\phi)x_t$. The former is the *term premium* component, as it drives the term premium, while the latter term is the *expectation* component, as it drives deviation of yields from the short-term rate that is due to expected changes in the short-term in the future. If the term premium component is negligible in size compared to the expectation component, then the term premium is zero for practical purposes. So, for example, if the correlation between the noise processes of consumption and the state variable (ρ_{cxt}) is zero, then the term premium is zero and the expectations hypothesis holds.[11] The formula also shows that there is no easy way to generate a sizeable term premium in this type of model, other than having high consumption volatility or high risk aversion. Indeed, it is not possible to generate a sizeable positive term premium in this class of models without a high risk aversion or a high consumption volatility. An earlier version of this article shows that this is the case by checking a variety of models. Models that incorporate habit in the utility function (Campbell and Cochrane 1999; Wachter 2006), can generate a high and positive term premium but there are states of the world in which effective risk aversion is exceedingly high.[12] In Section 5, it is also shown that models with recursive preferences also fail in this regard, unless the consumption volatility is high.

[10] Based on the Feynman-Kac formula, the equation can also be written like: $TP(x_t, m) = -\frac{1}{m} \log \left[\frac{\mathrm{E}_t^* \left[\exp \left\{ -\int_0^m r(x_{t+s}) \mathrm{d}s \right\} \right]}{\mathrm{E}_t \left[\exp \left\{ -\int_0^m r(x_{t+s}) \mathrm{d}s \right\} \right]} \right]$, where the numerator uses the risk-neutral measure.

[11] Indeed, it holds in its strongest form, as the term premium is exactly zero. In the literature, the expectations hypothesis usually refers to whether the term premium is changing over time, and not whether it is exactly zero.

[12] The earlier version of the article also includes a review of the bond premium puzzle literature.

2.8 Equity Pricing

Having defined the prices of bonds, a similar approach is used for stocks. First, stocks are assumed to pay a dividend that follows the process:

$$\frac{\mathrm{d}D_t}{D_t} = \mu_D \mathrm{d}t + \sigma_D \mathrm{d}W_{Dt} \tag{12}$$
$$\mathrm{d}W_{Dt}\mathrm{d}W_{ct} = \rho_{cD}\mathrm{d}t, \quad \mathrm{d}W_{Dt}\mathrm{d}W_{xt} = \rho_{xD}\mathrm{d}t$$

where D_t is the dividend flow at time t, μ_D is the drift of the dividend process, σ_D is the dividend diffusion, W_{Dt} is a Wiener process associated with the dividend process, ρ_{cD} is the correlation between the noises processes for consumption and the dividend, and ρ_{xD} is the correlation between the noises processes for the state variable and the dividend. For simplicity μ_D and σ_D are assumed to be constant.[13] Based on the dividend process, a dividend strip security can be constructed which pays a dividend at a specific point in time, and for each dividend strip we can define the strip price-dividend ratio, $\hat{s}_t(x_t, m)$, which according to Itô's Lemma follows the process:

$$\mathrm{d}\hat{s}_t = \left(\log(\phi)x_t\hat{s}_x - \hat{s}_m \mathrm{d}m + \frac{\sigma_{xt}^2}{2}\hat{s}_{xx}\mathrm{d}x_t^2\right)\mathrm{d}t + \sigma_{xt}\hat{s}_x\mathrm{d}W_{xt} \tag{13}$$

This is the ratio of the price of the dividend strip at time t (notice that at each point the current price is used and not the expected price at maturity) over the current dividend at time t. Thus, by definition the price of the dividend strip for $m = 0$ is 1. Having this clear terminal condition is useful, and is the reason why this approach is used.[14] Based on the terminal condition, the strip price-dividend

[13] The model can still be solved in a similar way if they depend on the state variable x_t.

[14] Trying to determine the price-dividend ratio for the stock directly gives an equation that does not have such a clear terminal equation. So, solving directly a differential equation of the price-dividend ratio requires the specification of two conditions Chen, Cosimano and Himonas (2010). As the alternative approach followed here shows, these conditions are not required to solve the problem.

ratio, $\hat{s}(x_t, m) = \hat{S}(x_t, m)/D_t$, is given by an equation similar to the bond price:[15]

$$
\begin{aligned}
&\mathrm{E}\big[\mathrm{d}(\Lambda\hat{S})\big] = 0 \\
\Rightarrow &\mathrm{E}\big[\mathrm{d}(\Lambda \underbrace{(\hat{s}D)}_{\substack{\text{price of}\\ \text{dividend}\\ \text{strip}}})\big] = 0 \Rightarrow \mathrm{E}\left[\frac{\mathrm{d}\Lambda}{\Lambda}\hat{s} + \mathrm{d}\hat{s} + \frac{\mathrm{d}D}{D}\hat{s} + \frac{\mathrm{d}\Lambda}{\Lambda}\mathrm{d}\hat{s} + \frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}D}{D}\hat{s} + \mathrm{d}\hat{s}\frac{\mathrm{d}D}{D}\right] = 0 \\
\Rightarrow &-\hat{s}_m - \underbrace{(r(x) - \mu_D + \rho_{cD}\gamma\sigma_{ct}\sigma_D)}_{\hat{r}(x_t)}\hat{s} + (\log(\phi)x_t - \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt} + \rho_{xD}\sigma_{xt}\sigma_D)\,\hat{s}_x + \frac{\sigma_{xt}^2}{2}\hat{s}_{xx} = 0 \\
&\hat{s}(x_t, 0) = 1
\end{aligned}
\tag{14}
$$

where D is the dividend paid at time $t + m$. The differential equation can be solved in the same way as the bond pricing equation. The risk premium of the dividend strip can also be derived from the pricing equation:[16]

$$
\begin{aligned}
&\mathrm{E}_t\left[\frac{\mathrm{d}(\hat{s}D)}{\hat{s}D}\right] + \mathrm{E}_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\right] = \underbrace{\mathrm{E}_t\left[\frac{\mathrm{d}(\hat{s}D)}{\hat{s}D}\right] - r(x_t)\mathrm{d}t}_{\equiv\text{Expected Excess Return}} = -\mathrm{E}_t\left[\frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}\hat{s}}{\hat{s}} + \frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}D}{D}\right] \\
&-\frac{\hat{s}_m}{\hat{s}} + \log(\phi)x\frac{\hat{s}_x}{\hat{s}} + \frac{\sigma_{xt}^2}{2}\frac{\hat{s}_{xx}}{\hat{s}} + \mu_D + \rho_{xD}\sigma_D\sigma_{xt}\frac{\hat{s}_x}{\hat{s}} - r = \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\frac{\hat{s}_x}{\hat{s}} + \rho_{cD}\gamma\sigma_{ct}\sigma_D
\end{aligned}
\tag{15}
$$

Next, by knowing the strip price-dividend ratio, for all maturities m, the stock price-dividend-ratio can be computed by integrating across all maturities:[17]

$$
\frac{S(x_t)}{D(x_t)} = s(x_t) = \int_0^\infty \hat{s}(x_t, m)\mathrm{d}m \tag{16}
$$

where the stock's dividend flow matches the dividends paid by the dividend strips.[18] The expected excess return of the stock can also be derived by integrating the

[15]This is slightly abusing notation. To be fully precise, an extra time subscript is required, ie $\hat{s}_t(x_t, m)$, in order to specify the timing of the dividend. In this notation, maturity occurs at time $t + m$. However, for brevity, maturity is assumed to be given and the time subscript is omitted.

[16]The risk premium is used equivalently as the expected excess return.

[17]A similar approach is used in Wachter (2006).

[18]In the case of a dividend strip, a lump sum amount is paid at a specific point in time, while the stock pays a dividend flow. For example, in the case of a dividend strip, if $D_0 = 1$ the dividend strip pays \$1 at time $t = 0$, and, in the case of a stock, if $D_t = 1$ for all $t \in (0, 1)$, then the stock pays a constant flow of dividends summing up to \$1 over the period from $t = 0$ to $t = 1$.

expected excess return of the dividend strip across all maturities:[19] [20]

$$\mathrm{E}\left[\frac{\mathrm{d}(sD)}{\underbrace{sD}_{\text{stock price}}}\right] + \frac{1}{s}\mathrm{d}t - r\mathrm{d}t = -\mathrm{E}\left[\frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}s}{s} + \frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}D}{D}\right] \quad (17)$$

$$-\frac{1}{s} + \log(\phi)x\frac{s_x}{s} + \frac{\sigma_{xt}^2}{2}\frac{s_{xx}}{s} + \mu_D + \rho_{xD}\sigma_D\sigma_{xt}\frac{s_x}{s} - r = \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\frac{s_x}{s} + \rho_{cD}\gamma\sigma_{ct}\sigma_D$$

3 Calibration

Time is measured in years for all relevant parameters.

The state variable is a steady-state-reverting process. The steady state is at $x_t = 0$. The speed of reversion is regulated by ϕ, which is set equal to 0.92. This also regulates the autocorrelation of the price-dividend ratio.[21] This implies that if $x_t = 1$, then x_{t+1} is expected to be roughly equal to 0.92 after a year. The state variable diffusion is set to $\sigma_{xt} = 0.2$. With this specification the state variable spends roughly 95% of the time in the interval (-1,+1).[22] So, these values are set equal to the left and right bounds respectively of the grid of the state variable.[23]

The time preference parameter is set to $\rho = 0.01$. The relative risk aversion parameter is set to $\gamma = 3$, which is within the common range used in the literature, and it is not considered too high.[24] The steady state consumption drift of the marginal investor is set to $\mu_{c0} = 0.12$. This was chosen in order to achieve the level of risk premia that we see in the data, but it is not implausibly high for a marginal investor. It is only slightly higher compared to the historic average return on stocks, and it is plausible to think that marginal investors achieve an even higher return by leveraging. After choosing the consumption drift, the steady

[19] Alternatively it can also be found directly through the pricing equation of the stock. Following Cochrane (2009) the pricing equation for a security with a dividend flow is given by $\mathrm{E}\big[\mathrm{d}(\Lambda S)\big] + \Lambda D\mathrm{d}t = 0$, where S is the price of the security, and D is the dividend flow. This equation can be rearranged to give the expected excess return.

[20] Here, we are integrating up to infinity, so $1/s$ is only included, which corresponds to the constant dividend payment. In the numerical solution a high cutoff is used, but still an extra term of the form $\hat{s}(x, M)/s$ is included, where M is the high cutoff maturity.

[21] This follows Wachter (2013).

[22] In the actual computations a larger grid is used but these are the bounds used to calibrate the interest rates, and also the bounds used in the plots of this article.

[23] The risk-free rate is not set to exact values at the bounds because targeting is not an exact process and requires trial and error.

[24] $\gamma = 2$ was also tried and it produces more or less similar results for risk premia. However, the value of 3 was chosen to make the consumption drift of the marginal investor more variable. In general, it is slightly curious that it is not easy to construct a non-highly volatile risk-free rate when either consumption drift or consumption diffusion are relatively high and significantly variable.

state consumption diffusion is set in such a way so that the risk-free rate at the steady state is equal to approximately 1%. This is close to historic values of the risk-free rate, as for example reported in Mehra (2007).[25] The targeting of the risk-free rate is done using Equation (6). This results in $\sigma_{ct} = 28.67\%$. This turns out to be a relatively high value, and even though this is not by construction, it fits the idea of the model. The reason is that intermediaries have their wealth invested in financial assets, and they could even be leveraged. So, high asset price volatility gets translated to consumption volatility. The same equation for the interest rate, is then again used to set the range of variation of the risk free rate to about 6% between the two bounds.[26] The chosen range roughly matches the level of variation over the last twenty-five years in the interest rate induced by monetary policy during tightening and loosening episodes. This exercise results in values for $\zeta = 0.07$ and $\eta = 0.055$.

In the dividend process, μ_D is set to 2.5%, and σ_D is set to 11% following Gabaix (2012).[27] The correlation between the dividend and consumption is set to $\rho_{cD} = 0.1$, which is relatively close to the value used in Campbell and Cochrane (1999).[28] The low value is also justified by the fact that in this model consumption changes are mostly associated with change in the state variable. In a similar vein, the correlation between the dividend and the state variable is likewise set to $\rho_{xD} = 0.1$. In general, the parameters chosen for the dividend process are not highly critical, to generate high risk premia, even though the dividend diffusion does play a small role in increasing the equity premium. In addition, it should be noted that the dividend process does not explicitly depend on the state variable. So, there are no effects from the state variable that are channeled through the dividend process to either the risk-free rate, the bond price, or the price-dividend ratio.[29] Finally, the correlation between the state variable and consumption is set to $\rho_{cxt} = 0.9$. This is a high value, and it is connected with the premise of the model that the state variable reflects investment opportunities. Unexpected changes in the state

[25]In that article, the estimates reported of the historic real risk-free rate for the USA range from 0.64% to 3.02%. However, for other countries negative values are also mentioned. For this reason, a target of 1% was chosen, which is at the lower end of estimates and close to the preferred estimate by Mehra and Prescott (1985), which is 1.31%

[26]The exact values are of the risk-free rate are 3.91...% and -2.51...% at the left and right bound of the state variable respectively.

[27]In that article, 2.5% was both the expected growth of the dividend and the expected growth of consumption. Here, it is only the expected growth of the dividends.

[28]In that article, a value of 0.2 is used.

[29]This is done for simplicity, in order to make the point of the model clearer, that the state variable acts through the discount channel. Allowing an explicit dependence of the dividend drift and diffusion on the state variable would in general not invalidate the results of this study.

variable lead to changes in the wealth and hence the consumption of the marginal investor.[30] This parameter can drive risk premia, as shown in the previous section. In addition, its sign determines the sign of the risk premia. Here the intuition is that as interest rates fall (for non-fundamental reasons), the marginal investor's wealth and hence her consumption increases. The same holds true for bonds and stocks, so both exhibit a positive risk premium.

4 Results

4.1 Risk-Free Rate

Before showing the results relating to risk premia, the behavior of the consumption process is shown, for different levels of the state variable. Figure 1 illustrates this behavior. For simplicity, the consumption drift and diffusion are shown compared to each other, and the state variable which is driving both quantities is not explicitly shown. This approach is used in the following figures as well. The consumption diffusion is shown in the x-axis. The range of the drift and the diffusion have been set in order to match the range of the risk-free rate. This leads to consumption drift exhibiting small variation, and consumption diffusion exhibiting moderate variation. Figure 2 shows the risk-free rate as a function of consumption diffusion. Given that the range of variation of the risk-free rate has been targeted through the calibration process, there is no risk-free rate puzzle by construction. Despite the absence of a risk-free rate puzzle, the model is still able to generate a high equity premium.

4.2 Equity

The results for average stock returns and average excess stock returns based on simulations are shown in Table 1. Despite its simplicity, the model matches both the unconditional moment of stock returns and stock excess returns. In both cases, the estimates deviate less than two standard errors from the historic averages. The standard errors are reported next to the historic averages, but they have been computed based on the model.[31] This model addresses the equity premium puzzle, and the only parameter that was used to amplify the equity premium is the

[30]The reader should keep in mind though that wealth is not explicitly modelled in the current article.

[31]From simulating 116 years multiple times and taking the standard deviation of the estimates. The prediction of the model is based on the average of multiple 116-year simulations.

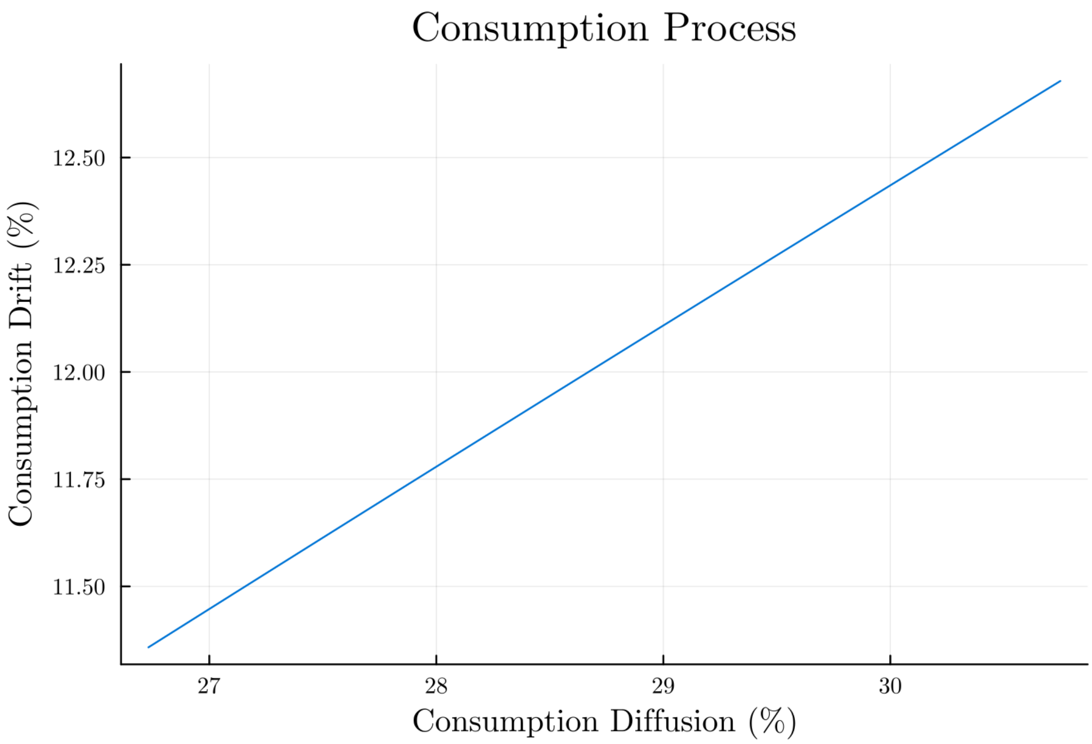

Figure 1: Different levels of consumption drift and diffusion, for different levels of the state variable. The range of variation in the drift and the diffusion has been set in order to target the variation of the risk-free rate.

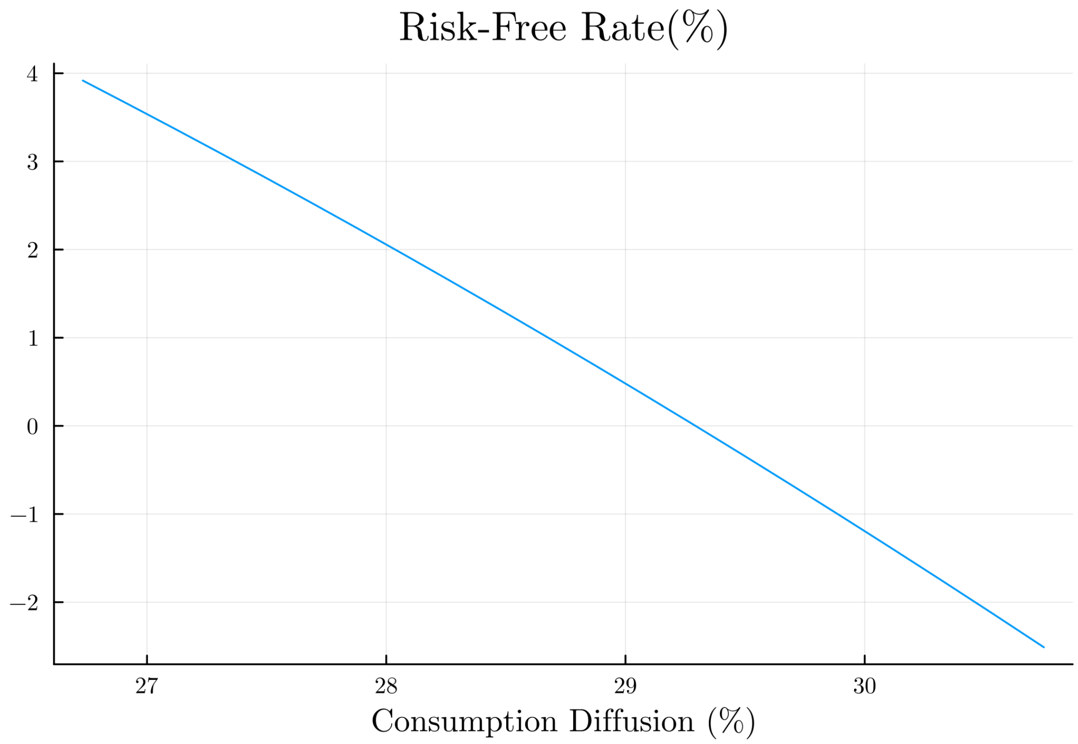

Figure 2: Annual risk-free rate as a function of consumption diffusion. The range of variation of the risk-free rate is targeted through the calibration process.

	Real Stock Return (%)		Risk-Free Rate (%)		Stock Risk Premium (%)	
	Average	St. Error	Average	St. Error	Average	St. Error
1899-2005 (Mehra-Prescott)	7.67	(1.09)	1.31	(0.66)	6.36	(1.21)
Model	5.86	-	0.94	-	4.88	-

Table 1: **Arithmetic Average USA Stock Returns in the data and in the model.**

consumption drift of the marginal investor, which is still kept at a plausible level.

Apart from the equity premium exhibiting a high level, it is also variable in time. This implies that apart from addressing the equity premium puzzle the model also exhibits predictability of returns. In particular, in the model, the equity premium level depends on the state variable. Thus, the state of the economy simultaneously drives current prices and future expected returns. Hence, it appears as though combinations of current prices or other economic variables can predict future excess returns. Table 2 shows that the log dividend-price ratio can predict future stock returns, and that predictability increases with horizon. Indeed, the model-implied slopes are close to the historic estimates.[32] The nature of the predictability is further shown in Figure 3, 4, and 5. These figures show the price-dividend ratio, the expected return of the stock, and the expected excess return as a function of the consumption diffusion of the marginal investor. The figures show that the price-dividend ratio can vary significantly for different levels of the consumption diffusion. In addition, in the model, the figures demonstrate that expected stock returns are more predictable than stock returns in excess of the short-term rate.[33]

4.3 Bonds

As was first shown by Backus *et al.* (1989), the bond premium puzzle suggests that long-term bonds have a positive risk premium that is significant in size. In addition, bond term premia have been found to be time-varying and predictable (Fama and Bliss 1987; Campbell and Shiller 1991; Cochrane and Piazzesi 2005). These results are based on nominal bonds, but these basic features have also been found to hold

[32]The R-squared values are not reported. As in the data, they are increasing with horizon, but they are generally lower in the model. This is not a big issue because the R-squared values can be amplified by choosing a lower value for σ_D, and this would not drastically change the other results.

[33]Unlike stock return predictability, the model does not generate enough stock return excess as in the data. The standard deviation of the log price-dividend ratio in the model is 0.045 whereas in the data based on Gabaix (2012) and Campbell (2003) it is 0.33.

	Data		Model
Horizon	Slope	St. Error	Slope
1 yr	0.11	(0.053)	0.138
4 yr	0.42	(0.18)	0.503
8 yr	0.85	(0.20)	0.791

Table 2: **Predictability of Stock Returns**
Following Gabaix (2012), this shows the estimates of the predictive regression for the stock return $r_{t\to t+T} = \alpha_T + \beta_T \log(D_t/P_t)$ over a specific horizon. Data: Campbell (2003).

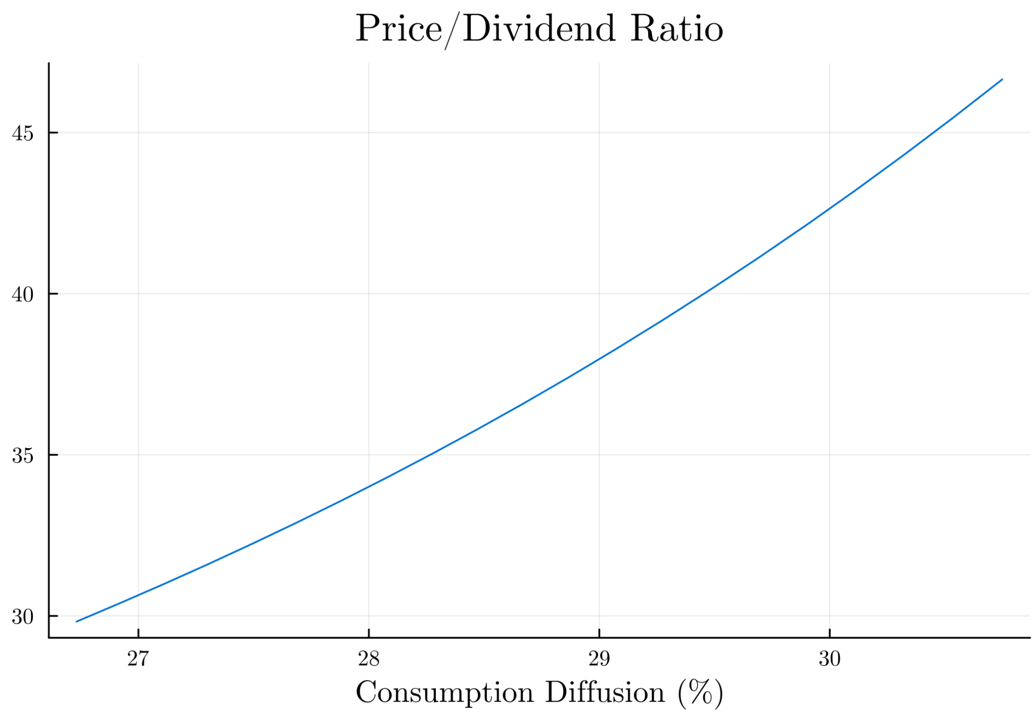

Figure 3: Price-Dividend Ratio of the stock as a function of the consumption diffusion of the marginal investor.

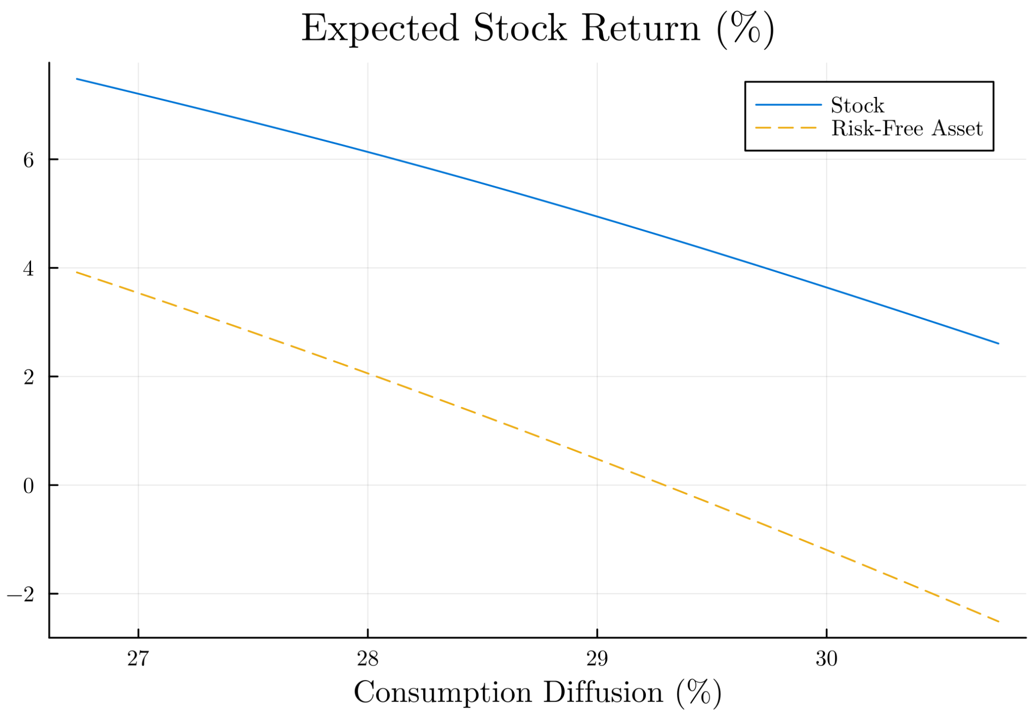

Figure 4: Expected Returns of the Stock as a function of the consumption diffusion of the marginal investor.

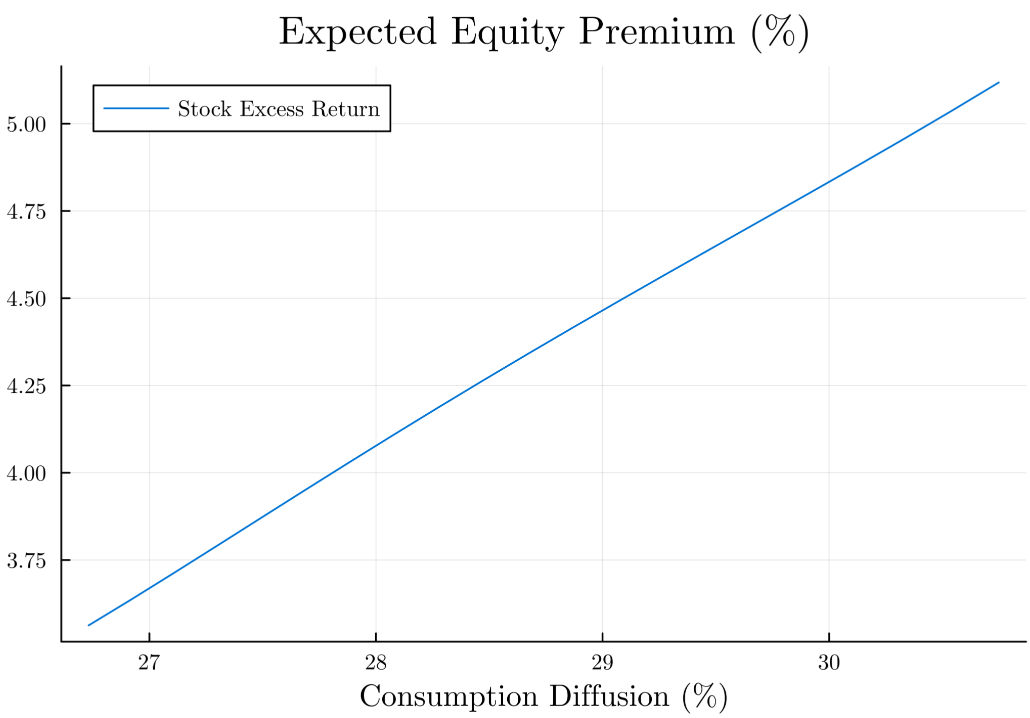

Figure 5: Expected Equity Premium as a function of the consumption diffusion of the marginal investor.

	20-Year Bond Risk Premium (%)	
	Average	St. Error
1926-2022 (Ibbotson)	2.30	(0.46)
Model	2.87	-

Table 3: **Arithmetic Average 20-Year US Government Bond Returns in the data and in the model. Source: Ibbotson Associates Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook.**

for real bonds (Abrahams *et al.* 2016; d' Amico *et al.* 2018; Pflueger and Viceira 2016). As is shown in Table 3, the average excess returns of long-term bonds are large and fit the data.[34] While the data may also contain an inflation risk premium because the bonds are nominal, in the model bonds are real. This is a significant result, because it seems to be harder to generate a positive and significant real excess bond return (or real term premium) compared to generating a high equity premium.[35] For example, long-run risk models are able to generate a high equity premium, but they do not generate a significant real bond risk premium. In this model, the marginal investor is (in the background) adjusting her portfolio to take advantage of changes in interest rates. In addition, her wealth and consumption react positively to drops in the interest rate, and this makes long-term bonds actually risky.[36] As a result, bonds exhibit a positive risk premium. Figure 6 shows the real yields of zero coupon bonds for different values of the state variable. The yield curve is upward sloping, and the term premium is in general positive. In addition, it is possible to see that longer-term bonds have have a significantly higher return compared to short-term bonds.

While the model captures the level of real bond risk premia, it does not fully deliver the predictability and the level of variation that we see in the data. Figure 7 shows the decomposition of the real ten-year bond into the term premium and the risk-neutral yield. The latter is by definition the difference between the yield and the term premium. As can be seen in the figure, the term premium varies for different levels of the state variable. However, it does not reach the level of

[34]The source of the historical data is the Ibbotson Associates Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook, as presented in Ross, Westerfield and Jordan (2024).

[35]Term premia and bond excess returns are not exactly the same, but they are closely related. Term premia are basically the same as excess returns, but they are measured over a longer horizon.

[36]While this is not the primary focus of this study, these consumption and wealth changes could indicate that marginal investors often have conflicting utility changes compared to the regular households in the economy.

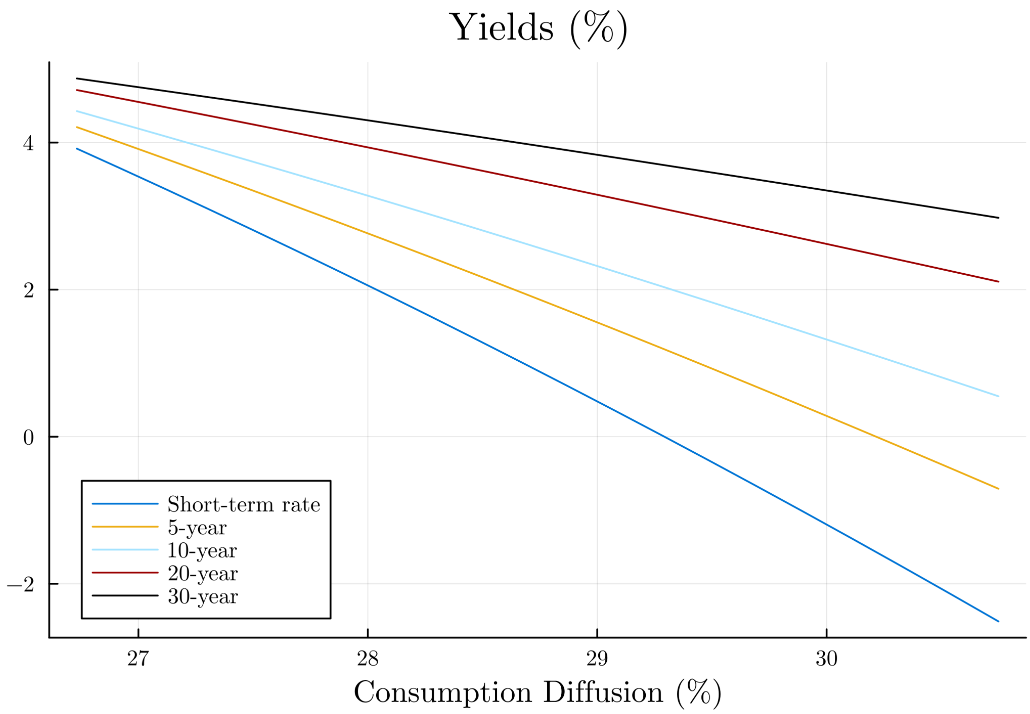

Figure 6: Real yields of zero-coupon bonds with various maturities.

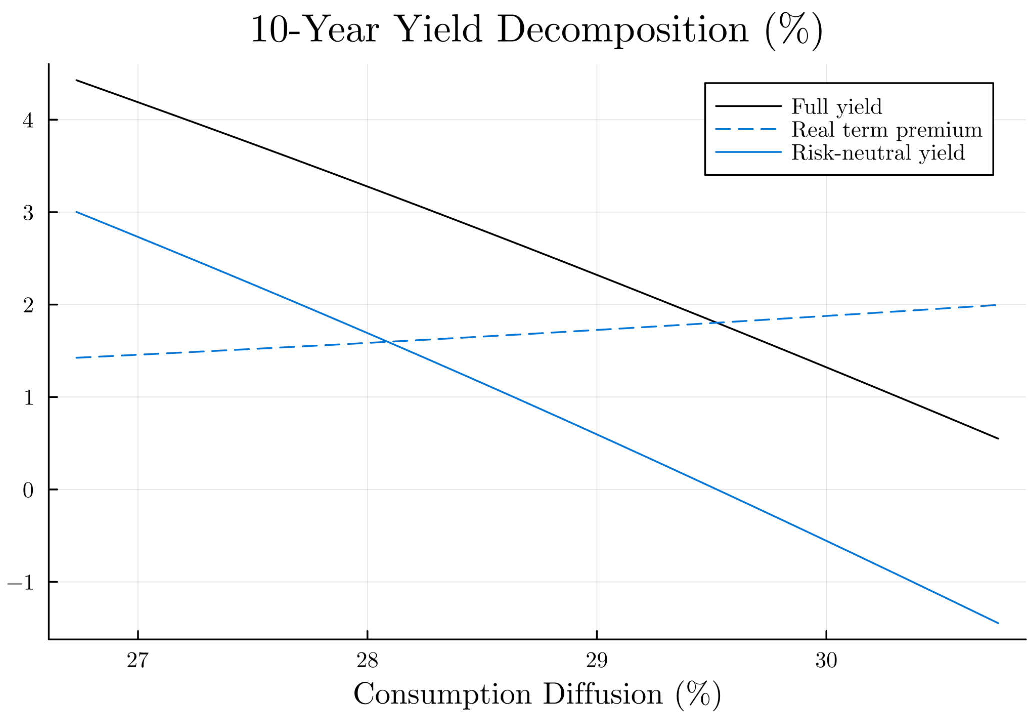

Figure 7: Decomposition of the real ten-year bond yield into the risk-neutral yield and the term premium.

variation that is found in Abrahams *et al.* (2016) and d' Amico *et al.* (2018).[37] While it would be favourable to be able to match the full level of variation, that we see in the data, it is not entirely surprising that this is not possible. In particular, we know that the yield curve can't be completely captured with just one state variable. So, the variability likely requires a slightly richer model. Arguably, this is not that difficult to deliver in a model with more state variables, because the harder part is to generate a positive and sizeable real bond term premium.

5 Recursive Preferences

In the main model of this article, CRRA utility is used. In this section, I discuss the case of recursive utility, to show that it does not generate high and positive term premia. Indeed this is significant, because there is literature that has shown it is possible to generate high equity premia with recursive utility (Bansal and Yaron 2004). These are the well-known long-run risk models.[38] Long-run risk models are characterized by a time-varying consumption drift, and possibly a time-varying consumption diffusion. Such models use a representative agent. So, consumption drift and the consumption diffusion take values that should be close to what we observe in aggregate data.

Following Duffie and Epstein (1992), when utility is recursive, the utility specification is given by:

$$V_0 = \mathrm{E}_0 \int_0^\infty F(C_t, V_t)\mathrm{d}t \tag{18}$$

Where V_t is the value function at time t, and $F(\cdot,\cdot)$ determines flow utility. The latter is also the aggregator of the recursive utility process, and it is given by:

$$F(C_t, V_t) = \frac{\rho\,(1-\gamma)\,V_t}{1-1/\psi}\left(\left(\frac{C_t}{((1-\gamma)\,V_t)^{-1/(1-\gamma)}}\right)^{1-1/\psi} - 1\right) \tag{19}$$

[37]As in the data, the model rejects the expectations hypothesis. However, the result of these regressions do not match the literature exactly. For example, while regressions as in Fama and Bliss (1987) give a relatively similar slope for low maturities as in that article, regressions as in Campbell and Shiller (1991) give estimates that are higher than 1 (estimates of 1 coincide with the expectations hypothesis and the article found negative estimates). Regressions as in Cochrane and Piazzesi (2005) can't be easily performed because the different forward variables are practically collinear, given that the model only has one state variable.

[38]As has been mentioned before, an earlier version of this article goes through many variations that show it is not possible to generate high and positive bond term premia.

Where ψ is the intertemporal elasticity of substitution. The case of CRRA utility is nested in this specification, and it corresponds to the case where $\psi = 1/\gamma$. The SDF is derived from the expressions for the value function and the aggregator function. As shown by Tsai and Wachter (2018), the value function can be expressed as:[39]

$$V_t = \frac{C_t^{1-\gamma} e^{(1-\gamma)K(x_t)}}{1-\gamma} \tag{20}$$

Where V_t increases with K, which is a specific function of x_t that captures the full dependence of the value function on the state variable.[40] At the end of this section, the expression above is justified, and a novel perturbation approximation that provides a formula for K is provided. Given the expression for the value function, Ito's Lemma can be implemented to get to the stochastic differential equation of the SDF. The calculation here follows Chen, Cosimano, Himonas and Kelly (2009). In particular, the fundamental relationship is:

$$\frac{\mathrm{d}\Lambda}{\Lambda} = F_V(C_t, V_t)\mathrm{d}t + \frac{\mathrm{d}F_C(C_t, V_t)}{F_C(C_t, V_t)} \tag{21}$$

F_C and F_V denote partial derivatives of F with respect to consumption and the value function respectively. The first term on the right-hand side is the derivative of the flow utility with respect to the value function. The second term can be computed by applying Ito's lemma on the derivative of flow utility with respect to consumption.[41] The result is the following:

$$\begin{aligned}\frac{\mathrm{d}\Lambda}{\Lambda} =& \Bigg(\frac{\rho(-(1-\gamma\psi)e^{\frac{(1-\psi)K[x_t]}{\psi}} - \gamma\psi + \psi)}{1-\psi} - \gamma\mu_{ct} + \frac{\gamma^2\sigma_{ct}^2}{2} + \frac{\gamma(\gamma\psi-1)\rho_{cxt}\sigma_{xt}\sigma_{ct}K'(x_t)}{\psi} \\ &+ \frac{(\gamma\psi-1)(-2\psi\log(\phi)x_t K'(x_t) + \sigma_{xt}^2((\gamma\psi-1)K'(x_t)^2 - \psi K''(x_t)))}{2\psi^2} \Bigg)\mathrm{d}t \\ &- \frac{(\gamma\psi-1)\sigma_{xt}K'(x_t)}{\psi}\mathrm{d}W_{xt} - \gamma\sigma_{ct}\mathrm{d}W_{ct}\end{aligned} \tag{22}$$

In the special case of $\gamma = 1/\psi$, which corresponds to time-separable utility, the equation above simplifies to the formula in Equation 5. Also, the stochastic component

[39] Similar results are common in the literature, see for example Benzoni, Collin-Dufresne and Goldstein 2011; Kraft, Seiferling and Seifried 2017.

[40] The model is not solved fully in this section because this is not required for the analysis. However, the earlier version of the article contains a full solution of the model. In addition, Melissinos (2023) provides a solution method based on a perturbation expansion of $K(\cdot)$.

[41] This operation is performed by substituting the value function using Equation (20) and applying Ito's lemma based on consumption and the state variable as independent variables.

relating to consumption ($-\gamma\sigma_{ct}\mathrm{d}W_{ct}$), is exactly the same as in the time-separable utility case, and there is an extra component, namely $-\frac{(\gamma\psi-1)\sigma_{xt}K'(x_t)}{\psi}\mathrm{d}W_{xt}$, due to the explicit dependence of the SDF on the state variable.

By using the expression for the SDF we can derive the risk-free rate, the stock price and the bond price, as was done in the main part of the article. Here we focus on the bond, in order to show that a positive and significant bond risk premium is not possible. In particular, the pricing equation for the bond is:

$$-B_m - r(x_t)B + \big(\log(\phi)x_t - \frac{(\gamma\psi-1)\sigma_{xt}^2 K'(x_t)}{\psi} - \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\big)B_x + \frac{\sigma_{xt}^2}{2}B_{xx} = 0 \tag{23}$$

The term premium is primarily driven by the "recursive" term, $-\frac{(\gamma\psi-1)\sigma_{xt}^2 K'(x_t)}{\psi}$. It turns out that the sign of this term is such that it produces a negative bond risk and term premium.[42] This holds in the standard case when recursive utility implies a preference for early resolution of uncertainty.[43] So, even though recursive utility can contribute to generating a high equity premium, and even though it can contribute in generating a high term premium in absolute value, it cannot generate a *positive* term premium. This shows that long-run risk models are not able to generate positive real term premia, even if recursive utility is used. So, while Bansal and Shaliastovich (2013) is able to generate positive nominal term premia by using an inflation channel, the real term premia in that article are still negative.

6 Conclusion

This article has systematically analysed the performance of a model with respect to basic macrofinance puzzles, such as the equity premium puzzle, the bond premium puzzle, and the predictability of stock returns. The model has used a single state variable that drives investment opportunities, which are simultaneously affecting the consumption drift and the consumption diffusion of the marginal investor. This is consistent with the intermediary asset pricing approach to asset pricing, in which marginal investors are not representative consumers. While a lot of puzzles can be addressed in this framework, the excess volatility in stock prices and the

[42]The reason for this is that the sign of this term is the same as the sign of the derivative of the short-term rate, which in turn determines the sign of B_x. Thus, the full term in the differential equation always has the same sign.

[43]This can also be seen in the formula, as early resolution of uncertainty is the condition that $\gamma > 1/\psi$.

predictability of bond term premia are not fully captured.

The implications of this perspective are significant. Firstly, if it is only possible to generate realistic asset price movements by assuming that marginal investors are not representative consumers, this means that intermediaries, or marginal investors, in general are not investing according to the preferences of regular households. In addition, it means that they are able to extract excess returns on average from financial markets in exchange for bearing risk. If regular households are not able to participate in financial markets due to institutional frictions or barriers, this implies that marginal investors are able to extract rents from financial markets. Secondly, it is critical for the conduct of monetary policy, because it implies that movements in term premia have a limited connection to the consumption of regular households. So, for instance, when the central bank tries to decrease long-term bond yields, this may be mostly affecting the economic situation of a small group of investors and not so much the overall economic situation, which central banks typically try to affect.[44]

In further research, it would be interesting to extend the model to capture asset pricing features more fully. Extensions in this direction could include the introduction of a second state variable, to try to capture more asset pricing movements. In addition, it would be interesting to use the model as a tool to understand historic movements in asset prices. Finally, the marginal investors that are modelled in this article are best understood as a subset of agents, who are particularly active in financial markets. Other agents are not explicitly modelled. In further research, it would be interesting to explicitly model the other agents, in order to understand why regular households do not seem to fully participate in financial markets, and how the ordinary households are affected by movements in asset prices and by the behavior of marginal investors.

[44]Vayanos and Vila (2021), after a comment by John Cochrane, also make the point that monetary policy affecting the short-term rate can be viewed as a source of arbitrageur rent.

A Feynman-Kac Formula

The differential equations arising from the pricing equations for the bond and the strip price-dividend ratio are solved using the Feynman-Kac formula.

A.1 Feynman-Kac Formula for the Bond Price

Equation 9 is repeated here:

$$-B_m - r(x_t)B + \underbrace{\left(\log(\phi)x_t - \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt}\right)}_{\tilde{\mu}(x_t)} B_x + \frac{\sigma_{xt}^2}{2}B_{xx} = 0$$

The Feynman-Kac formula states that the solution to this equation is given by:

$$B(x_t, m) = \mathrm{E}\left[\exp\left(-\int_0^m r(\tilde{x}_{t+s})\mathrm{d}s\right) \Big| \tilde{x}_t = x_t\right] \tag{24}$$

$$\mathrm{d}\tilde{x}_t = \tilde{\mu}(\tilde{x}_t)\mathrm{d}t + \sigma_{xt}(\tilde{x}_t)\mathrm{d}W_{xt} \tag{25}$$

where $\tilde{\mu}(\cdot)$ is defined by the term multiplying B_x above.

A.2 Feynman-Kac Formula for the Strip Price-Dividend Ratio

Equation 14 is repeated here:

$$-\hat{s}_m - \underbrace{\left(r(x) - \mu_D + \rho_{cD}\gamma\sigma_{ct}\sigma_D\right)}_{\hat{r}(x_t)} \hat{s} + \underbrace{\left(\log(\phi)x_t - \rho_{cxt}\gamma\sigma_{ct}\sigma_{xt} + \rho_{xD}\sigma_{xt}\sigma_D\right)}_{\hat{\mu}(x_t)} \hat{s}_x + \frac{\sigma_{xt}^2}{2}\hat{s}_{xx} = 0$$

The Feynman-Kac formula states that the solution to this equation is given by:

$$\hat{s}(x_t, m) = \mathrm{E}\left[\exp\left(-\int_0^m \hat{r}(\hat{x}_{t+s})\mathrm{d}s\right) \Big| \hat{x}_t = x_t\right] \tag{26}$$

$$\mathrm{d}\hat{x}_t = \hat{\mu}(\hat{x}_t)\mathrm{d}t + \sigma_{xt}(\hat{x}_t)\mathrm{d}W_{xt} \tag{27}$$

where $\hat{r}(\cdot)$ is defined by the term multiplying $\hat{s}_x$ above and $\hat{\mu}(\cdot)$ is defined by the term multiplying $\hat{s}_x$ above.

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